7. Wave Motion & Stationary Waves

- Continuous disturbance that transfers energy without any net displacement of the medium particles is called wave.

- Different types of waves on the basis of their production are mechanical wave, electromagnetic wave and matter wave.

- Different types of mechanical waves on the basis of their propagation transverse wave and longitudinal waves.

- Mechanical waves need a medium for its propagation.

- Representation of a sinusoidal wave travelling along the positive x-axis is

  $y(x, t) = a \sin(kx - \omega t + \Phi)$

- The equation can also be represented as linear combination of sine and cosine function.

  $y(x, t) = A \sin(kx - \omega t) + B \cos(kx - \omega t)$, where $a = \sqrt{A^2 + B^2}$ and $\Phi = \tan^{-1}\left(\frac{B}{A}\right)$

- **Superposition Principle**: The net displacement is the vector sum of the displacements caused by individual waves at that point.

  $Y = Y_1 + Y_2 + \ldots + Y_n$ (For wave grouping)

- **Interference** is the redistribution of energy when two waves with a constant phase difference interact.
  - Constructive interference: Net displacement is maximum.
  - Destructive interference: Net displacement is minimum.
- **Reflection of Wave**
  - A wave, whether transverse or longitudinal, while travelling in a certain medium undergoes a change in phase when it is incident on the boundary of another medium.
  - A wave travelling in a rarer medium suffers a change in phase by straight pi radians when it is incident on the boundary of a denser medium.

- For a wave travelling in a denser medium like water, there is practically no resistance when it is incident on the boundary of a rarer medium like air.

- **Quincke's tube experiment** provides a good laboratory method of measuring the velocity of sound in air.

- The fundamental frequency of the vibrations in a stretched string,

  f=12LTμ.

- **Law of Length**

  The frequency of a vibration produced by a stretched string is inversely proportional to its length.

  Thus, $f \propto \frac{1}{l}$.

- **Law of Tension**

  The frequency of a vibration is directly proportional to the square root of the tension in a stretched string.

  Thus, $f \propto \sqrt{T}$.

- **Law of Mass**

  The frequency of a vibration is inversely proportional to the square root of the mass per unit length of the stretched string.

  Thus, $f = \frac{1}{\sqrt{M}}$.

- **Node** is a point on the vibrating string, which has the maximum tension and the least displacement.
- **Anti-node** is the point where the displacement is maximum and tension zero.
- **Stationary Waves**

In strings, stationary waves produce frequencies in multiples of v2l or harmonics of v2l i.e
v=nv2l=n2lTμ

- **Stationary waves:** In strings, stationary waves produce frequencies multiple of $\left(\frac{v}{2l}\right)$ or harmonics of $\left(\frac{v}{2l}\right)$, i.e.

$$\nu = \frac{nv}{2l} = \frac{n}{2l}\sqrt{\frac{T}{\mu}}$$

- **Closed pipe:** In closed pipes, only odd harmonics are produced, i.e $\nu = \frac{(2n+1)v}{4l}$, with fundamental frequency of $\left(\frac{v}{4l}\right)$.

- **Open pipe: In** open pipes, all harmonics with fundamental or first harmonic $\left(\frac{v}{2l}\right)$ are produced, i.e. $v=\frac{nv}{2l}$, where v is the velocity of sound.
- **Beats:** Beats arise when two waves with slightly different frequencies, n_1 and n_2, and comparable amplitudes, are superposed. The beat frequency is $\nu_{beat}=|\nu_1-\nu_2|$.

End correction

- The end correction is numerically expressed as e =0.3 d.

Cause of end correction

- The cause of end correction is that the air particles in the plane of the open end of the tube are not free to move in all directions.

Calculation of End Correction

- **When a pipe is closed at one end:**

  e = n1l1 - n2l2n1-n2

- **When a pipe is open at both ends:**

  e = n1l1 - n2l22n1-n2

Limitations of End Correction

- Inner diameter of the tube must be uniform throughout the length.
- Effects of air outside, and that of the temperature of the air outside, are to be neglected.
- The tuning fork must be held in such a way that the tip of its prong must be horizontal, at the centre and at a small distance above the open end of the tube.

Doppler Effect

- Doppler Effect: It is the change in pitch of a sound when there is relative motion between the sound source and the observer.

  ν=V±V0V±Vsν0

$+V_0$ if observer approaches the source
$-V_0$ if observer recedes from the source
$-V_s$ if observer approaches the observer
$+V_s$ if observer recedes from the observer

Applications of Doppler's effect:

- Doppler's effect is used to measure the velocities of moving objects in diverse areas such as military, medical science, astrophysics, etc.
- It is also used by police to check over-speeding of vehicles.
- Doppler shift, an application of Doppler's effect, is used at airports to guide aircraft and in the military to detect enemy aircraft.
- In astrophysics, Doppler's effect is used to measure the velocities of stars and planets.

- Doctors use it to study heart beat and blood flow in different parts of the body.

Limitations of Doppler's effect in sound

- It is applicable when the velocities of the sources of sound and observer are much lower than the velocity of sound
- The motion of source and the observer must be along the same straight line.
- The medium must be in rest; otherwise, the formula has to be modified.

- **Forced oscillation** → When an external agency maintains an undamped oscillation by compensating for the loss of energy, it is called forced oscillation. The external force is a sinusoidal force.
- The expression for the external force is given by $F = F_m \sin(\omega_d t)$
- Here, F_m is amplitude of external force and ω_d is driving frequency
- The displacement of the natural oscillation dies out according to $x(t) = A\cos(\omega_d t + \Phi)$.
- The Amplitude, A, is the function of the forced frequency (ω_d) and the natural frequency, ω and is given by
  A equals F subscript m over open curly brackets m squared open parentheses omega squared minus omega subscript d squared close parentheses plus omega subscript d squared b squared close curly brackets to the power of begin display style 1 half end style end exponent
- Cases of damping:
- **Case 1:** Small damping; driving frequency far from natural frequency

  omega subscript d b less than less than m open parentheses omega squared minus omega subscript d squared close parentheses therefore A equals fraction numerator F subscript m over denominator m left parenthesis omega squared minus omega subscript d squared right parenthesis end fraction

- **Case 2:** Driving frequency close to natural frequency ω_d is very close to ω.
  m open parentheses omega squared minus omega subscript d squared close parentheses less than less than omega subscript d b therefore A equals fraction numerator F subscript 0 over denominator omega subscript d b end fraction

- **Resonance**: The phenomenon of increase in amplitude when the frequency of the driving force is close to the natural frequency of the oscillator is called resonance.

$$\omega' \approx \omega_0$$

- The principle behind the phenomenon of resonance finds application in stethoscopes and in the tuners of radio sets.
- Resonance is used to increase the intensity of sound in musical instruments and to analyse musical instruments.
- The unknown frequency of a vibrating tuning fork can be determined using resonance.
- In string instruments, sound is produced by the vibration of strings.
- Sitar, veena, guitar and tanpura are examples of string instruments.
- In wind instruments, sound is produced by the vibration of air columns.
- Flute, bassoon and harmonium are examples of wind instruments.
- In percussion instruments, sound is produced by setting vibrations in a stretched membrane.
- Mridangam, tabla and drums are some examples of percussion instruments.